Hyperlinks are shown in blue, download the `cdf player` *from the Wolfram Alpha website to view the* `Wolfram Alpha interactive demonstrations`*. When you have downloaded the cdf player, click on this symbol to view the demonstration.*

Sets (Section 6.1) Continued

Union of 3 sets If A and B and C are sets, their union $A \cup B \cup C$ is the set whose elements are those objects which appear in at least one of A **or** B **or** C.

Example If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$, list the elements of the set $A \cup B \cup C$.

$A \cup B = \{1, 2, 3, 4, 6, 8\}$, $(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 8\}$.

$B \cup C = \{2, 3, 4, 5, 6, 8\}$, $A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 8\}$.

$A \cup C = \{1, 2, 3, 4, 5, 6\}$, $B \cup (A \cup C) = \{1, 2, 3, 4, 5, 6, 8\}$

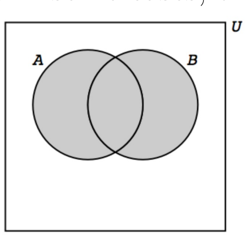
Example If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$ are subsets of the universal set $U = \{1, 2, 3, \ldots, 10\}$, list the elements of the set $A^c \cup B \cup C$.

$A^c = \{5, 6, 7, 8, 9, 10\}$, $B \cup C = \{2, 3, 4, 5, 6, 8\}$, $A^c \cup (B \cup C) = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

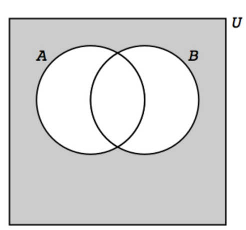
Example Give a verbal description of the set $R \cup E \cup C$ from our class survey example. Are you in this set?

Give a verbal description of the set $(R \cup E \cup C)' = (R \cup E \cup C)^c$. Are you in this set?

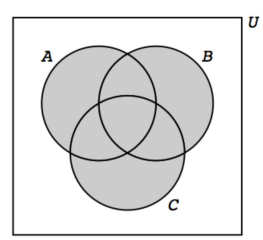
Venn Diagrams We can also draw representations of two and three subsets of a universal set using Venn diagrams as shown below. The shaded regions below represent the given subsets of the universal set. (Note that in some cases, there are two set theoretic descriptions of the same set.)



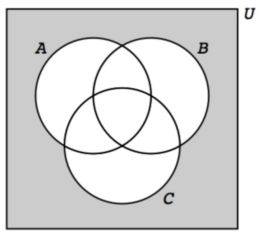
$A \cup B$
$(A' \cap B')'$



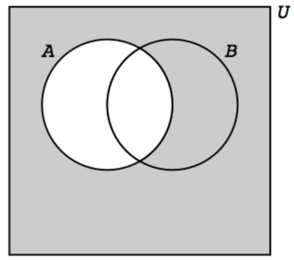
$A' \cap B'$
$(A \cup B)'$



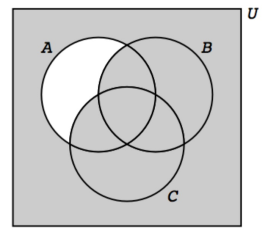
$A \cup B \cup C$
$(A' \cap B' \cap C')'$



$A' \cap B' \cap C'$
$(A \cup B \cup C)'$



A'



$A' \cup B \cup C$
$(A \cap B' \cap C')'$

Note To find the shaded region corresponding to the union of two sets on a Venn diagram, one can shade both sets individually and the the resulting shaded region corresponds to the union of the two sets.

The following interactive Venn diagram applet on Wolfram Alpha will allow you to experiment with identifying shaded regions of Venn diagrams:

Intersection If A and B are sets , Then $A \cap B$, read A intersection B, is a new set. Its elements are those objects which are in A **and** in B i.e. those elements which are in both sets.

Example If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$, list the elements of the set $A \cap B$.

$\{2, 4\}$

If A and B and C are sets, their intersection $A \cap B \cap C$ is the set whose elements are those objects which appear in A **and** B **and** C i.e. those elements appearing in all three sets.

Example If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. List the elements of the set $A \cap B \cap C$.

$A \cap B = \{2, 4\}$ so $A \cap B \cap C = \{4\}$.

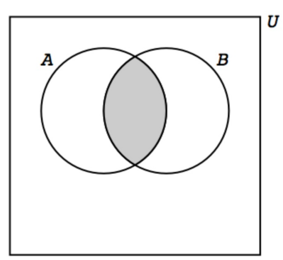
Example If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$ are subsets of the universal set $U = \{1, 2, 3, \ldots, 10\}$, list the elements of the set $A' \cup (B \cap C)$.

$A' = \{5, 6, 7, 8, 9, 10\}$, $B \cap C = \{4, 6\}$ so $A' \cup (B \cap C) = \{4, 5, 6, 7, 8, 9, 10\}$

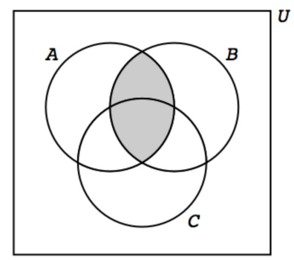
Example Give a verbal description of the people in $R \cap E \cap C$, where R, E and C are the sets described in our class survey example. Are you in this set?

Give a verbal description of those in the set $R' \cap E$. Are you in this set?

The following **Venn diagrams** show shaded regions corresponding to some intersections. In general, to find the shaded region corresponding to two sets A and B, you should shaded the sets A and B in different colors and the set $A \cap B$ will be the region where both shadings(colors) occur.



$A \cap B$
$(A' \cup B')'$



$A \cap B$
$(A' \cup B')'$

For representations of more intersections open the applet .

Disjoint sets Two sets A and B are said to be disjoint if $A \cap B = \emptyset$. For example if $A = \{$All major league baseball players who got more than 700 home runs in their career$\} = \{$ Barry Bonds, Hank Aaron, Babe Ruth$\}$ and $B = \{$All major league baseball players with a career batting average greater than .350$\} = \{$ Ty Cobb, Rogers Hornsby, Joe Jackson$\}$, then A and B are disjoint, i.e. $A \cap B = \emptyset$.

Note The empty set has the following properties: For any set A,

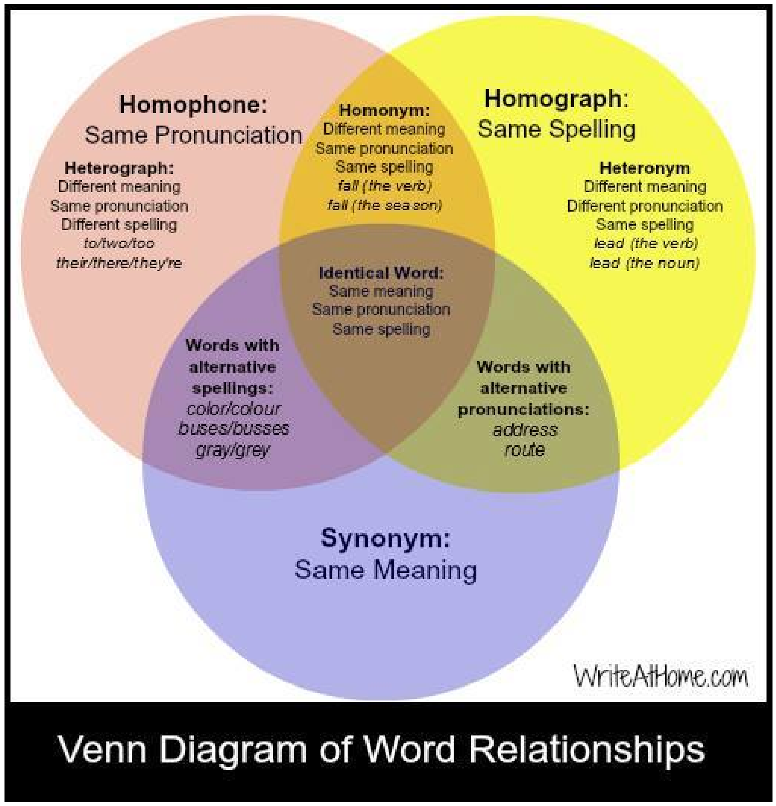
$$\emptyset \cup A = A, \quad \emptyset \cap A = \emptyset \quad \text{and} \quad \emptyset \subset A.$$

Note Complements have the following properties:

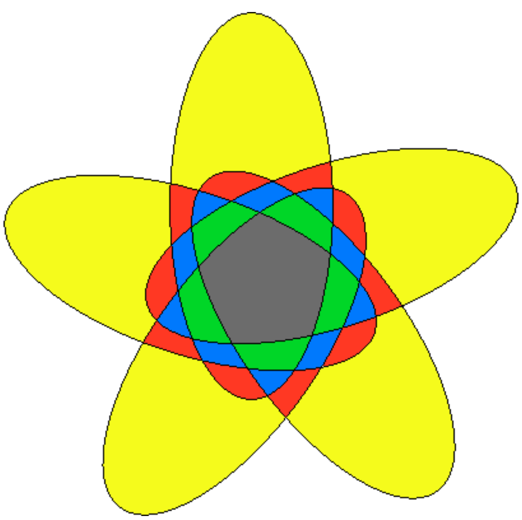
$$A \cap A' = \emptyset, \quad (A')' = A \quad A \cup A' = U$$

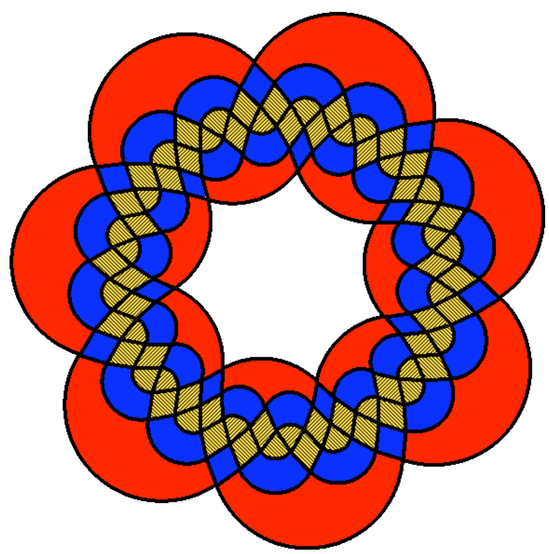
Verify these properties for example (*) above.

Venn diagrams of two or three sets are often used in presentations. Venn diagrams of more sets are possible, but tend to be confusing as a presentation tool because of the number of possible interactions.



The following diagrams show Venn diagrams for five sets on the left and for 7 sets on the right.





Counting Elements in a subset using a Venn Diagram (Section 6.2)

The Inclusion-Exclusion Principle

Definition For any finite set, S, we let $n(S)$ denote the number of objects in S.

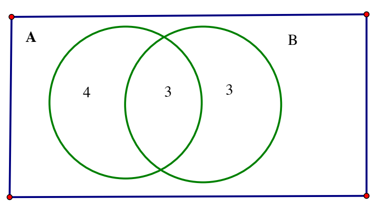
The Inclusion Exclusion Principle If A and B are sets, Then

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}.$$

Example Check that the Principle is true for the following sets:

$$A = \{1, 2, 3, 4, 5, 6, 7\}, \qquad B = \{5, 6, 7, 8, 9, 10\}$$

We can use a Venn diagram showing the number of elements in each basic region to display how the numbers in each set are distributed among its parts.



Thus we can sometimes use the inclusion-exclusion principle either as an algebraic or a geometric tool to solve a problem.

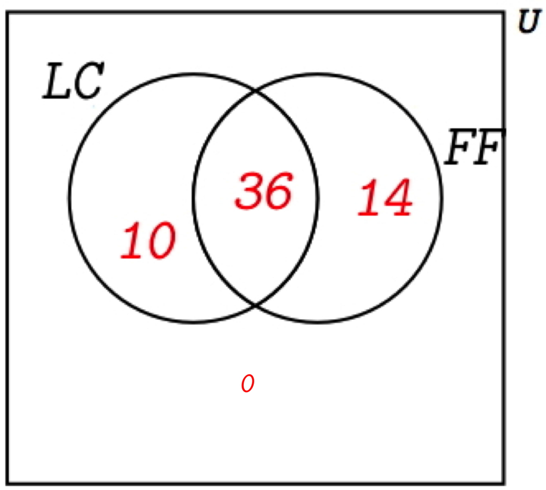
Example Let A and B be sets, such that $n(A) = 10$ and $n(B) = 12$ and $n(A \cup B) = 15$, then how many elements are in the set $A \cap B$?

Example Let A and B be sets, such that $n(A \cup B) = 20$, $n(B) = 10$ and $n(A \cap B) = 5$, then how many elements are in the set A? (Solve this using both methods: algebra and a Venn diagram)

Example A survey of a group of students, revealed that 60 of them liked at least one of the cereals, Frosted Flakes or Lucky Charms. If 50 of them liked Frosted Flakes and 46 of them liked Lucky Charms,
(a) How many of them liked both cereals?

(b) Draw a Venn diagram showing the results of the survey.

$n(\text{FF}) = 50$; $n(\text{LC}) = 46$; $n(\text{FF} \cup \text{LC}) = 60$.
Hence $n(\text{FF} \cap \text{LC}) = 50 + 46 - 60 = 36$.



(c) How many students liked Frosted Flakes but did not like Lucky Charms?

Note that if two sets A and B do not intersect, then $n(A \cap B) = 0$ and hence $n(A \cup B) = n(A) + n(B)$. Now apply this to a set and its complement to get

$$\boxed{n(A) + n(A') = n(U)}$$

where U is the universal set.

Example A survey of 70 students revealed that 64 of them liked to learn visually. How many of them did not like to learn visually?

Example 68 students were interviewed about their music preferences. 66 of them liked at least one of the music types, Rap, Classical and Eighties. How many didn't like any of the above music types?

Example (A combination of the two principles) In a survey of 70 students on Movie preferences, the students were asked whether they liked the movies "The Breakfast Club" and "Ferris Bueller's Day Off"., (All students had seen both movies and the only options for answers were like/dislike), 50 of the students said they liked "The Breakfast Club" and 25 of them said they didn't like "Ferris Bueller's Day Off". All students liked at least one of the movies.

(a) How many students said they liked both movies?

(b) Display the survey results on a Venn diagram.

Answer

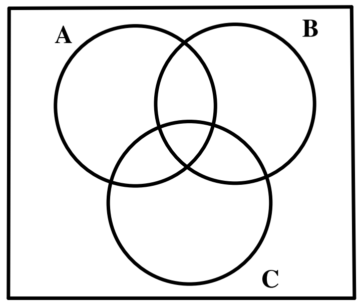
Example In a survey of a group of 70 movie-goers, 62 liked the movie "Catching Fire", 42 liked the movie "Divergent" and 39 liked both movies.

(a) Represent this information on a Venn Diagram.

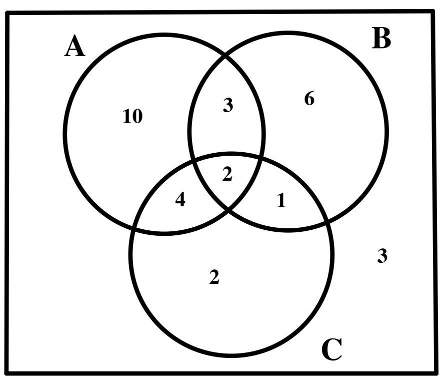
Answer

(b) Use the Venn diagram to find how many of those surveyed did not like either movie.

Venn Diagrams of 3 sets. A Venn diagram of 3 sets divides the universal set into 8 non-overlapping regions. We can sometimes use partial information about numbers in some of the regions to derive information about numbers in other regions or other sets.



Example The following Venn diagram shows the number of elements in each region for the sets A, B and C which are subsets of the universal set U.



Find the number of elements in each of the following sets:

(a) $A \cap B \cap C$

(b) B'

(c) $A \cap B$

(d) C Answer

(e) $B \cup C$

Example In a survey of a group of 68 Finite Math students, 62 liked the movie "The Fault in our Stars", 42 liked the movie "The Spectacular Now" and 55 liked the movie "The Perks of Being a Wallflower". 32 of them liked all 3 movies, 39 of them liked both "The Fault in Our Stars" and "The Spectacular Now", 35 of them liked both "The Spectacular Now" and "The Perks of Being a Wallflower" and 49 of them liked both "The Fault in Our Stars" and "The Perks of Being a Wallflower". Represent this information on a Venn Diagram.

Answer

Example In a survey of a group of 68 Finite Math students (Spring 2006), 50 said they liked Frosted Flakes, 49 said they liked Cheerios and 46 said they liked Lucky Charms. 27 said they liked all three, 39 said they liked Frosted Flakes and Cheerios, 33 said they liked Cheerios and Lucky Charms and 36 said they liked Frosted Flakes and Lucky Charms. Represent this information on a Venn Diagram. How many didn't like any of the cereals mentioned?

Example The results of a survey of 68 Finite Math students(Spring 2006) on learning preferences were as follows: 64 liked to learn visually, 50 liked learning through listening and 36 liked learning Kinesthetically. 21 liked using all three channels, 47 liked to learn visually and through listening, 35 liked to learn both visually and kinesthetically, 21 liked to learn through listening and kinesthetically. How many preferred only visual learning?

Old Exam questions for Review

1 In a group of 30 people, 15 run, 13 swim, 13 cycle, 5 run and swim, 8 cycle and swim, 9 run and cycle, and 5 do all three activities. How many of the 30 people neither run nor cycle?

(a) 8 (b) 10 (c) 9 (d) 12 (e) 11

2 Out of 50 students who exercise regularly, 25 jog, 20 play basketball and 15 swim. 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim and 2 people do all three. How many students do not do any of these activities?

(a) 10 (b) 15 (c) 4 (d) 0 (e) 2